The Study on Direct Adaptive Fuzzy Controllers

Shun-Feng Su, Juan-Chih Chang, and Song-Shyong Chen

Abstract

Direct adaptive fuzzy controllers have been proposed and discussed in the literature. Even though such controllers have been proven to be effective, in our study, some phenomena are observed. In this paper, those problems of using adaptive fuzzy controllers for unknown nonlinear systems are reported. The role of a parameter matrix required in the Lyapunov equation is discussed. It can be found that even though the Lyapunov synthesis approach has already proven that the system is asymptotically stable, the parameter matrix still needs to be selected appropriately beside of the symmetric positive definite property. Another issue is that it can also be found in our simulations that this kind of adaptive fuzzy controllers does not converge to a fixed controller as assumed in the literature, but adaptively adjusts its parameters according to the errors. As a consequence, when the considered system has sensory noise, the system may gradually become unstable. Ways of restraining such an unbounded phenomenon are proposed. From our simulations, the proposed approaches can have nice performance.

Keywords: Adaptive Fuzzy Control, Lyapunov Theorem, Stability.

1. Introduction

Linguistic fuzzy models are excellent in handling uncertain information and can also be proven to be universal approximators [1]-[3]. In fact, fuzzy systems have demonstrated good performance in various real applications as reported in the literature [25], [26]. Fuzzy control has been a very active research field in the fuzzy community and also has shown promising results in various applications [9]-[12], [27]. Adaptive fuzzy control [7], [17]-[24] is an important methodology in fuzzy control. Even though adaptive fuzzy control has been proven to be effective, in our study, some phenomena are observed. Two kinds of adaptive fuzzy control can be found in the literature. In adaptive fuzzy control, an adaptive control law is derived to ensure the convergence of the adaptive control. If such an adaptive law is for the controller itself [7], this controller is referred to as a direct adaptive fuzzy controller in the literature. On the other hand, if the adaptive law is to model certain terms in the considered systems [18], the obtained controller is referred to as an indirect adaptive fuzzy controller. In this study, some phenomena in direct adaptive fuzzy control are observed and reported.

In the fuzzy control analysis process, usually, the Lyapunov stable condition is employed to guarantee the system stability [4]-[7]. The Lyapunov theorem [8] is to define an energy-like Lyapunov function and the theorem states that the system is asymptotically stable when the Lyapunov function is convergent. In the design process of using fuzzy adaptive controllers for nonlinear systems, the Lyapunov stable condition is also employed to guarantee the stability of a system [7]. In our study of fuzzy adaptive controllers for nonlinear systems, we found that even though the Lyapunov theorem has guaranteed the stability of the control system, if certain parameters are not properly selected, the system may still become unstable. We shall discuss this issue in this paper.

Another issue reported in the paper is that it can also be found in our simulation that the controller does not converge to a fixed value but varies with trajectories. It is because when there exist errors, the controller will change those parameters according to the derived adaptive law. This seems contraction to what has been claimed in the literature [7]. This changing behavior stimulates our interest in the following problem. Can the adaptive controller survive when there exist sensory errors? In our simulation, it can be found that when the adaptive controller is employed for an unstable system with sensory noise, the output error will drive the control

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Manuscript received 20 May, 2006; accepted 28 September, 2006.

unbounded. Thus, if no constraints are used, the system will gradually become unstable. In this paper, two ways of restraining controllers are proposed to resolve this problem. Simulations show that both approaches can resolve the unstable problem.

2. Direct Adaptive Fuzzy Controllers

Fuzzy control is a control of process through fuzzy linguistic descriptions. Recently, it has been successfully applied to many practical problems [9]-[12]. Usually, fuzzy controllers can achieve better control performances than conventional controllers do. Control design may face uncertainties occurring in systems or even unknown systems. Thus, it is strongly desired to have adaptive fuzzy controllers. In the following, the derivation of the direct adaptive fuzzy controller is introduced. Basically, the introduction of the direct adaptive control in this section is proposed in [7]. In [7] and other similar adaptive fuzzy control approaches, the membership functions are always assumed to be fixed and the consequences of fuzzy rules are viewed as adjustable parameters. In this study, we also assume the same situation. Thus, the adaptive process considered is to identify those parameters used to define the consequences of fuzzy rules.

Now consider an nth-order nonlinear system of the form

$$x^{(n)} = f(x, \dot{x}, ..., x^{(n-1)}) + bu \qquad y = x \tag{1}$$

where f is an unknown continuous function, b is a positive unknown constant, and $u \in R$ is the input of the system, and $y \in R$ is the output of the system. Assume that the state vector $\underline{x} = (x_1, x_2, ..., x_n)^T = (x, \dot{x}, ..., x^{(n-1)})^T \in R^n$ is measurable. The control objective is to force y to follow a given bounded reference signal y_m. Hence, it is to determine a feedback control u, which is a constructed from a tunable parameter vector $\underline{\theta}$. If $\underline{\theta}$ can be adjusted appropriately, the feedback control u can approach the optimal feedback control $u*$. Define the tracking error as $e \equiv y - y_m$. When e approaches zero, the nth-order nonlinear system can track the reference signal well. If the nth-order nonlinear system is known (the continue function f and the positive constant b are known), then the so-called perfect feedback control is

$$u* = \frac{1}{b}[-f(\underline{x}) + y_m^{(n)} + \underline{c}^T \underline{e}] \tag{2}$$

where $\underline{e} = (e, \dot{e}, ..., e^{(n-1)})^T \in R^n$ and $\underline{c} = (c_1, c_2, ..., c_n)^T \in R^n$. When the coefficients $c_1, c_2, ..., c_n$ are properly chosen, then it implies that $\lim_{t\to\infty} e(t) = 0$.

In practice, it is difficult to know the system equation, especially for complicated systems. When f and b are unknown, the perfect control $u*$ cannot be obtained. The adaptive fuzzy controller proposed in [7] is to develop a way of approaching this optimal feedback control. The idea is to adaptively approach the perfect control by a fuzzy system. The used fuzzy system is to perform a mapping from the current states to the desired input u. The mapping consists of a set of fuzzy IF-THEN rules. In our study, the l-th rule is of the form

$$R^{(l)}: \textbf{IF } x_1 \text{ is } F_1^{(l)} \text{ and ... and } x_n \text{ is } F_n^{(l)} \quad \textbf{THEN } u = \theta^{(l)}, \tag{3}$$

where x_1, …, x_n are the state variables, $F_1^{(l)}$, …, $F_n^{(l)}$ are the corresponding fuzzy labels, and $\theta^{(l)}$ is the corresponding output value for the l-th rule. By using the product operations for the conjunction relations in the premise parts of fuzzy rules, the output of a fuzzy system consisting of N rules is obtained as:

$$u = \frac{\sum_{l=1}^{N} \theta^{(l)} (\prod_{i=1}^{n} \mu_{F_i^{(l)}}(x_i))}{\sum_{l=1}^{N} (\prod_{i=1}^{n} \mu_{F_i^{(l)}}(x_i))} = \theta^T \xi(x) \tag{4}$$

where $\mu_{F_i^{(l)}}(x_i)$ is the membership degree of x_i belonging to the fuzzy label $F_i^{(l)}$, $\theta = [\theta^{(1)}, \theta^{(2)}, \cdots, \theta^{(N)}]^T$, and $\xi = [\xi^{(1)}, \xi^{(2)}, \cdots, \xi^{(N)}]^T$ and is referred to as the regressive vector. Here the supscript T for a vector is the transpose of the vector, and $\xi^{(l)} = \frac{\prod_{i=1}^{n} \mu_{F_i^{(l)}}(x_i)}{\sum_{l=1}^{N} (\prod_{i=1}^{n} \mu_{F_i^{(l)}}(x_i))}$ is called the fuzzy basis function [1].

Now, define the optimal parameter vector $\underline{\theta}*$ as

$$\underline{\theta}* = \arg \min_{\theta \in R^n} [\sup_{x \in R^n} | u_c(\underline{x} | \underline{\theta}) - u* |] \tag{5}$$

and the minimum approximation error is

$$\omega = u_c(\underline{x} | \underline{\theta}*) - u* = \underline{\theta}*^T \underline{\xi}(\underline{x}) - u*. \tag{6}$$

Also, define $\underline{\Theta} = \underline{\theta}* - \underline{\theta}$. The desired adaptive law can be derived in a Lyapunov stability sense.

Define a Lyapunov function as [7]

$$V = \frac{1}{2} \underline{e}^T P \underline{e} + \frac{b}{2\gamma} \underline{\Theta}^T \underline{\Theta} \tag{7}$$

where γ is a positive constant, and P is an $n \times n$ positive symmetric definite matrix [8][13]. Let $\underline{p}_n$ be the last column of P. From [7], the adaptive law is obtained as

$$\underline{\dot{\theta}} = \gamma \underline{e}^T \underline{p}_n \underline{\xi}(\underline{x}) \tag{8}$$

With such an adaptive law, $\dot{V} = -\frac{1}{2}\underline{e}^T Q \underline{e} - \underline{e}^T PB\omega \le 0$ and the Lyapunov theorem states that the system is stable. In the above derivation, it is required that $|\underline{e}^T PB\omega| < \frac{1}{2}\underline{e}^T Q\underline{e}$. As indicated in [1], when there are enough rules to describe a fuzzy system, the minimum approximation error ω will be very small. It is quite reasonable to assume that $|\underline{e}^T PB\omega| < \frac{1}{2}\underline{e}^T Q\underline{e}$. Hence, $\dot{V}$ is negative. Therefore, it can be claimed that the system is stable [7].

3. Study of Direct Adaptive Fuzzy Controllers

In this session, we shall report our observations about employing the above adaptive fuzzy controller for two different systems. The first one is the system used in [7] and the second one is a drastic unstable system. The first example considered is a first-order unstable system [7] described as

$$\dot{x}(t) = \frac{1-e^{-x(t)}}{1+e^{-x(t)}} + u(t) \tag{9}$$
$$y = x$$

It is easy to verify that the system is unstable if there is no control input. The control objective is to regulate the output of the system to the origin. The same as that in [7], six fuzzy sets are defined over the interval [–3, 3] with membership functions as $\mu_{NL}(e) = 1/(1+\exp(5(e+2)))$, $\mu_{NM}(e) = \exp(-(e+1.5)^2)$, $\mu_{NS}(e) = \exp(-(e+0.5)^2)$, $\mu_{BS}(e) = \exp(-(e-0.5)^2)$, $\mu_{BM}(e) = \exp(-(e-1.5)^2)$, and $\mu_{BL}(e) = 1/(1+\exp(-5(e-2)))$. Those membership functions are also shown in Fig. 1 for illustration. The following values are used; $x(0)=1$, $\gamma=1$, b=1, the sampling rate is 0.01, and $\theta_i(0)'s$ are zeros.

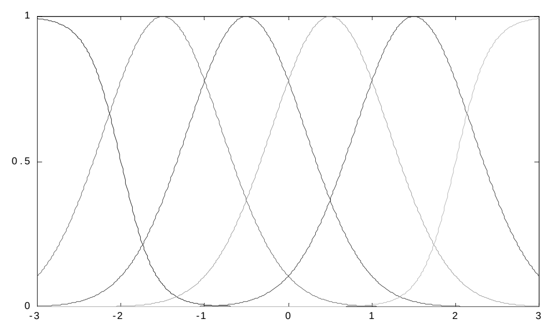
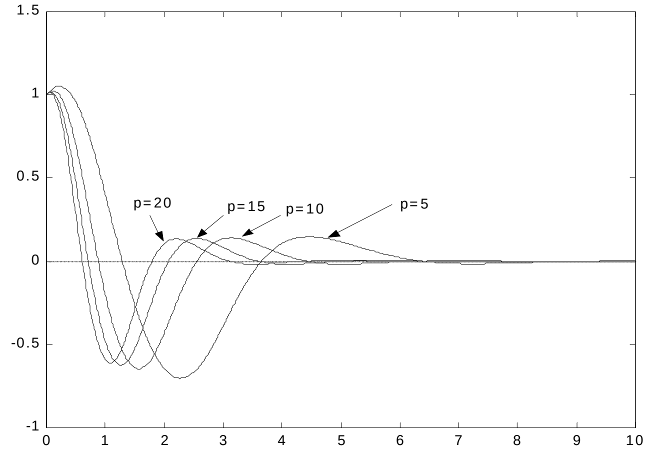
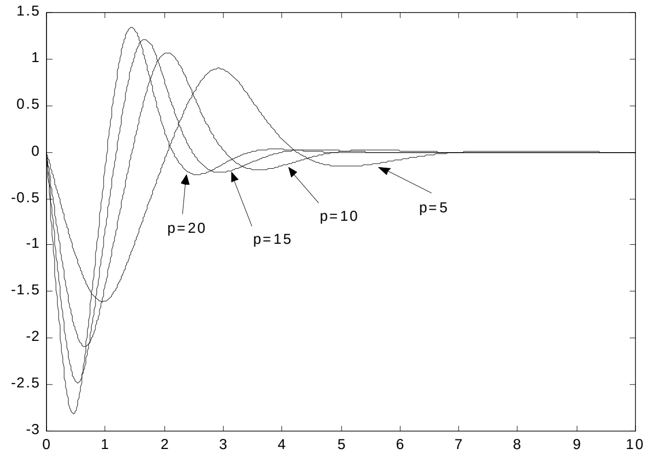


Figure 1. Fuzzy membership functions defined over the state space.

First, we want to study the effects of using various values for the parameter *P*. It should be noted that *P* is a positive definite matrix used in the Lyapunov function (Eq. (7)). Since the considered system is a first-order system, *P* becomes a scalar. Different values are used here (*P*=5, 10, 15, and 20, respectively). Figures 2 are the simulation results. Fig. 2 (a) shows the actual output y and desired output y_m, and Fig. 2 (b) shows the control u for these cases. It can be found that when *P* is large, the speed of the convergence is fast and the value of the control u is also large. It seems no problem here. Nevertheless, it can be found that in the next example, when *P* is large, the system may become unstable.



(a)



(b)

Figure 2. (a) The actual output y and the desired output y_m, and (b) the control u, with the different *P*=5, 10, 15, 20.

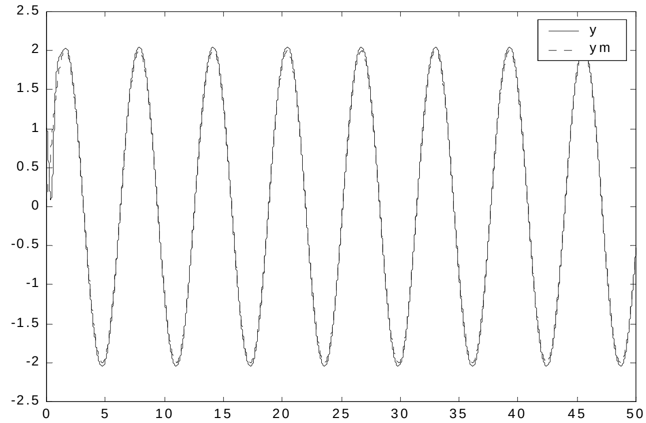
Now, the control objective is to control the actual output y of the unstable system to track a desired trajectory y_m=2sin(t). The used parameters are $x(0)=1$, $\gamma=1$, b=1, and the initial $\theta_i(0)'s$ are zeros. In order to achieve better tracking performance, we set *P*=100. The simulation results are given in Figures 3. Fig. 3 (a) shows the the actual output y and the desired output y_m. Fig. 3 (b) is the control u, and Fig. 3 (c) is the adaptive behavior of the parameter $\underline{\theta}$. In the derivation of the adaptive fuzzy control in [7], it is assumed that there exists an optimal control as (2). However, from the above simulation results, it can be found that the control seems not convergent to a fixed values or to a limit cycle. Instead, the trajectories seem divergent. From the adaptive law, the parameters are tuned if errors exist. In other words, the system will attempt to change its control law whenever there exist errors. The controller is to adaptively tune the parameters to reduce errors found instead of using the found controller to eliminate errors. This can be seen in our simulation that when the system is trained for a period of time and we stop the adaptive mechanism, the system become unstable. In other words, the learned controller cannot control the system.

The second example considered is also a first-order unstable system, but it is more drastically unstable than the system is in Example 1. The considered system is

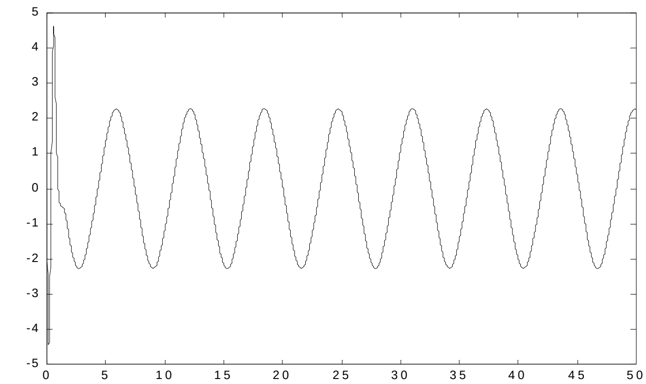
$$\begin{aligned} \dot{x} &= e^{x} - 1 + u \\ y &= x \end{aligned} \qquad (10)$$

The main objective of the controller is to regulate the output of the drastic unstable system to the origin. The initial values for x, $\underline{\theta}$, γ, and six membership functions are the same as that used in previous example.

Now, the performances of using various *P*'s are again observed. Fig. 4 (a) shows the results for using *P*=10. It can be seen that the direct adaptive fuzzy controller fail to control the system. Fig. 4 (b) shows the result of using *P*=11. Now, the controller can control the drastic unstable system to the origin. In our simulations, it can be found that the system can converge for $P \geq 11$, but diverge for $P \leq 10$ in initial $x(0) = 1$. In other words, there exists a lower bound for *P*. It should be noted that the derivation in section 2 states that as long as *P* is positive the system will be stable in a Lyapunov sense. However, from this result, it is easy to find that the proposed adaptive algorithm cannot stabilize the system when *P* is small. From the adaptive law, it can be found that *P* serves as a role like a learning constant in a traditional learning algorithm. If *P* is small, it means the tuning effect is small. But, in this example, the system possesses drastic behavior. Thus, if the tuning amount is too small to match up the drastic behavior of the system, the system will certainly go unstable. In conclusion, when we design a direct adaptive fuzzy controller, the parameter *P* of the Lyapunov equation (7) must be selected appropriately; otherwise, the controller cannot have stable control for unstable systems. Finally, we use the different initial $x(0)$ =0.5, 0.6, 0.7, 0.8, 0.9, 1 to test the same system. The corresponding *P*'s that will make the system stable or unstable are tabulated in Table 1.



(a)



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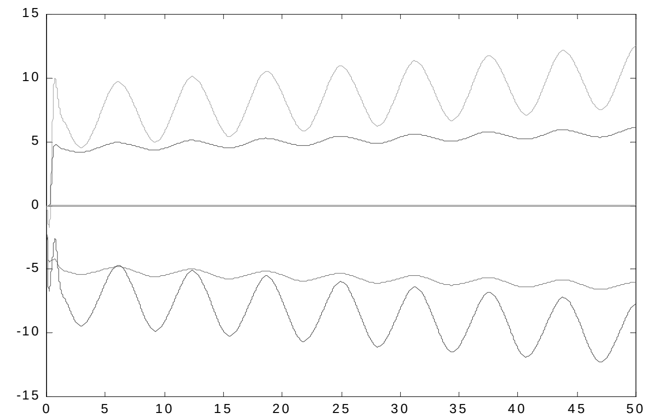
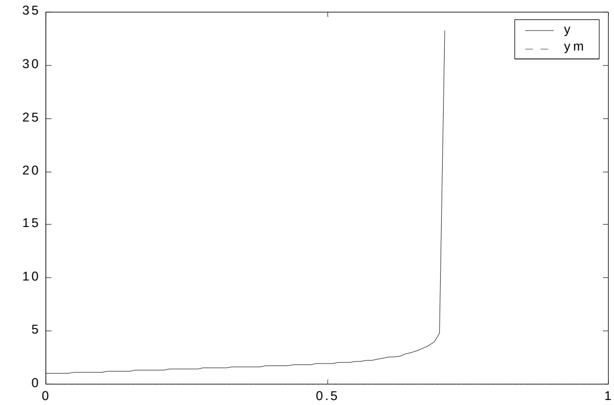
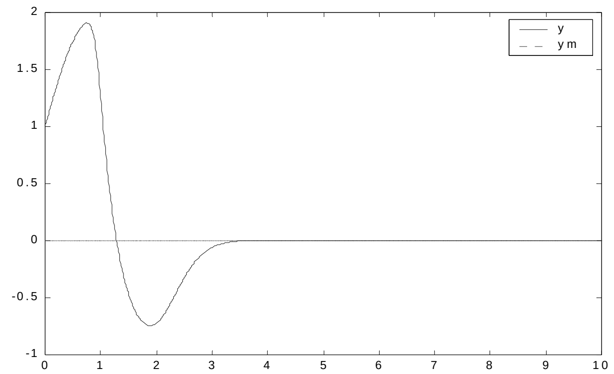


(c)

Figure 3. (a) The actual output y and the desired output y_m, (b) the control u, and (c) the value of the $\underline{\theta}$ of the consequence with six membership functions.



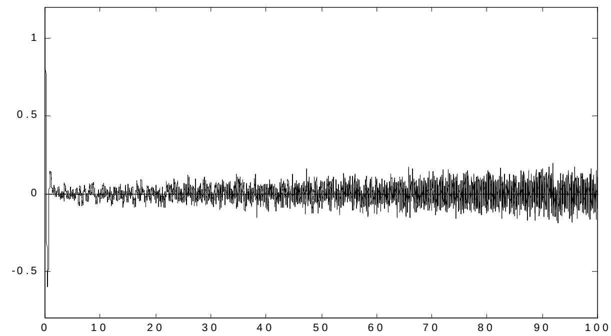
(a)



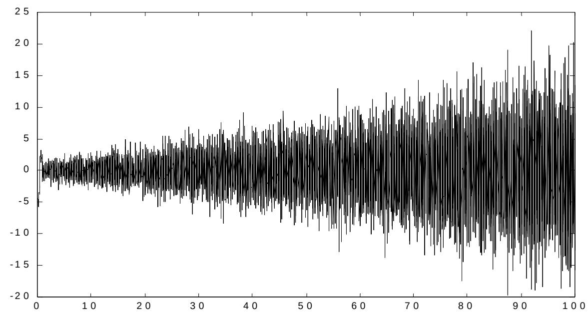
(b)

Figure 4. (a) The actual output y is divergent for P=10 and (b) convergent for P=11.

From the above discussion, it can be seen that if there are errors, the system will tune the parameters. Now we consider a real scenario in which there exist sensory errors; say r_n, which is called an external noise in this study. Now, the measured error is $e = y_m - y - r_n$. The regulation problem (i.e., to drive the output of this first-order system to the origin y_m=0) and the tracking problem (i.e., to track a desired trajectory $y_m = 2\sin(t)$) are both considered. First, consider the first example and assume that P=100, the external noise $r_n = 0.1randn(1)$, and the other parameters are the same as those used in the above simulation for Example 1. It can be seen that the system cannot converge. Figures 5 (a) – (d) show the output *y*, the control *u*, the error *e*, the tuned parameters, respectively, for the regulation case. Figures 6 (a) – (d) show the results for the tracking case. It is evident that due to the existence of a nonzero error, there is a variation value in the adaptive law. Then, the parameter θ is tuned by the variation value. Such a phenomenon can be seen in Fig. 5 (d) and Fig. 6 (d). Since the value θ is related to the control *u*, the control *u* is then divergent. Fig. 5 (b) and Fig. 6 (b) show the behavior.



(a)



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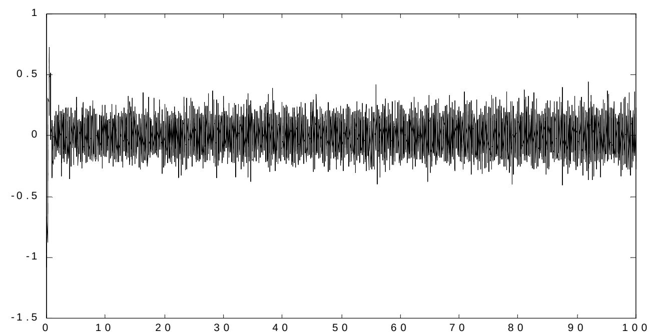

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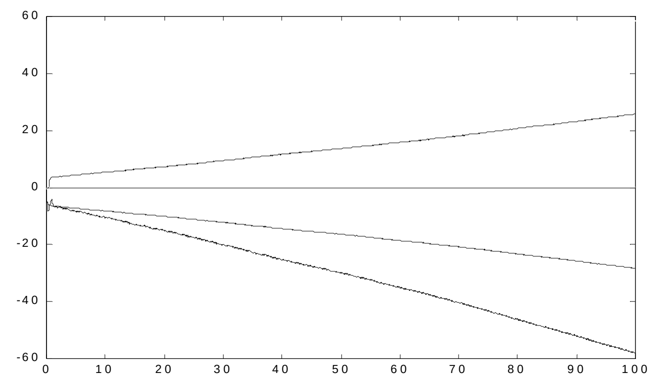
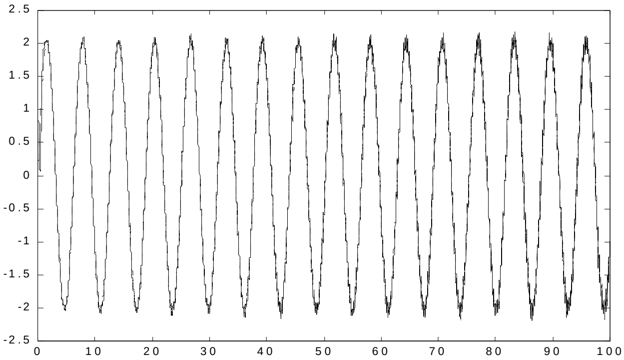


(d)

Figure 5 (a) The actual output *y*, (b) the control *u*, (c) the error *e*, and (d) the six parameter θ of the simple fuzzy controller, with $r_n = 0.1randn(1)$, P=100, and no bound.



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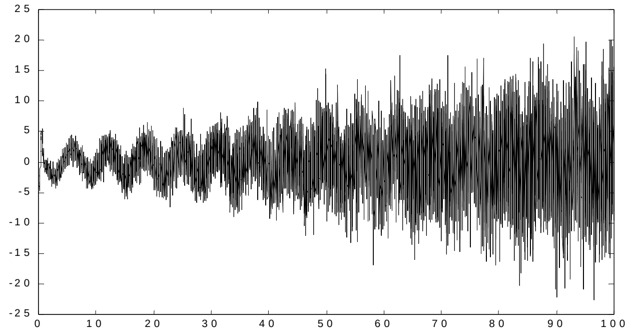

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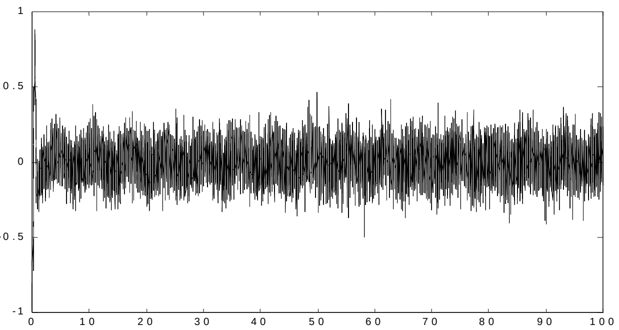

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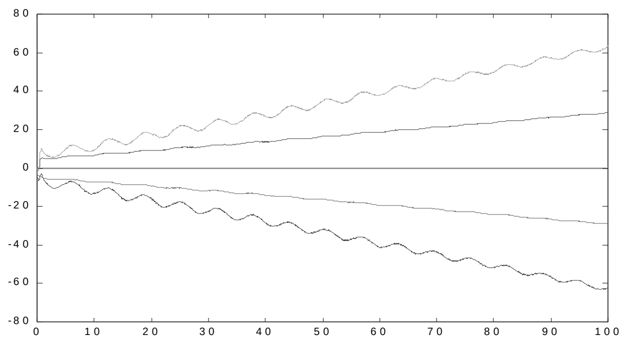
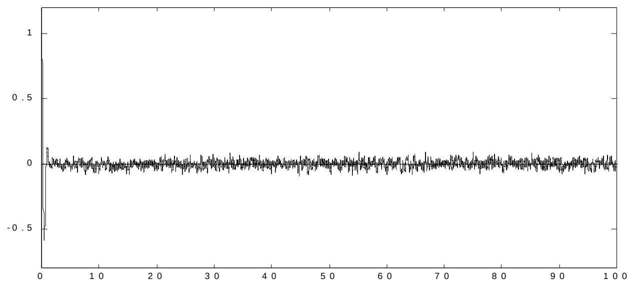


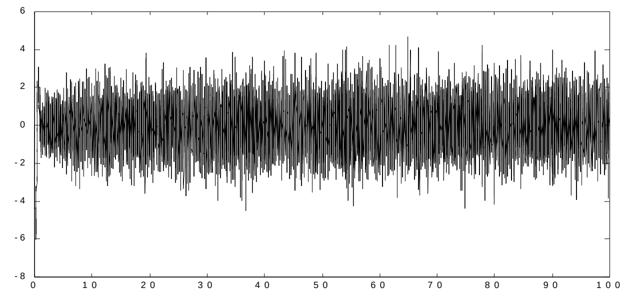
(d)

Figure 6 (a) The actual output *y* and the desired output y_m, (the control *u*, (c) the error *e*, and (d) the six θ of the pu fuzzy controller with $r_n = 0.1randn(1)$, *P*=100, and r bound.

Since the divergence behavior is driven by unbounded control, which is owing to the unbounded tuning for those parameters, a simple way of resolving this problem is to constrain those parameters. Two methods are proposed to constrain those parameter tuning. This first one is to set a bound M_θ for all parameters. The idea is that if all parameters θ's do not exceed the bound M_θ, then the parameters can be tuned accordingly. If some of the parameters exceed M_θ, these parameters are set to be equal to the bound M_θ and the others still follow the adaptive law to adjust. In the simulation, we set $M_\theta = \pm 10$. Figures 7 (a) – (d) show the output *y*, the control *u*, the error *e*, and the six tuned parameters, respectively for the regulation case and Figures 8 (a) – (d) show the results for the tracking case. From those simulations, it is evident that the controller can indeed achieve the desired goal in some extend. However, from Fig. 7 (d) and Fig. 8 (d), it can be found that the controller eventually becomes a bang-bang controller. Thus, we attempt to resolve this situation by another approach.



(a)



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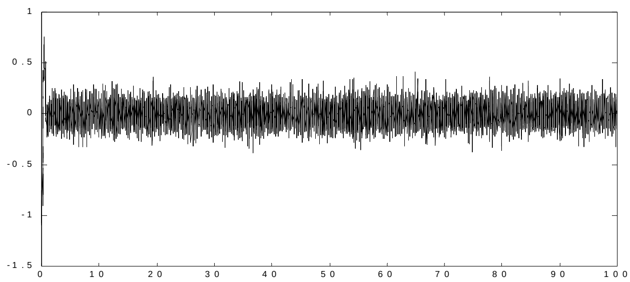

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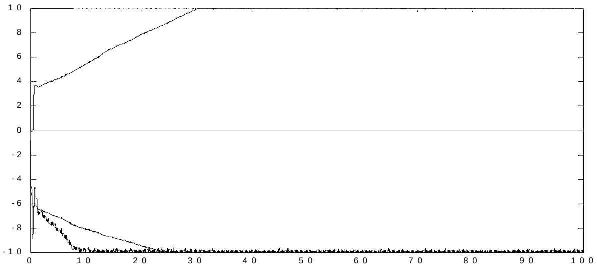


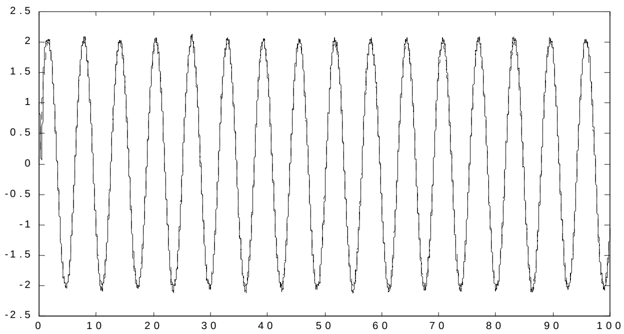
(d)

Figure 7 (a) The actual output *y*, (b) the control *u*, (c) the error *e*, and (d) the six parameters θ of the pure controller with $r_n = 0.1randn(1)$, *P*=100, and $M_\theta = \pm 10$.

Now, we define a bound $M_{n\theta}$ for the norm of θ instead of for each parameter. The idea is similar to the above except that if the norm of θ exceeds the bound $M_{n\theta}$, the norm of θ is shrunk to be equal to $M_{n\theta}$. The update parameters θ can be calculated as follows:

$$\theta_{new} = \theta_{old} * \frac{M_{n\theta}}{|\theta_{old}|} \tag{11}$$

In the simulation, we set $M_{n\theta} = \pm 10$. Figures 9 (a) – (e) show the output y, the control u, the error e, the tuned parameters, and the norm of the parameters, respectively for the regulation case. Figures 10 (a) – (e) show the results for the tracking case.



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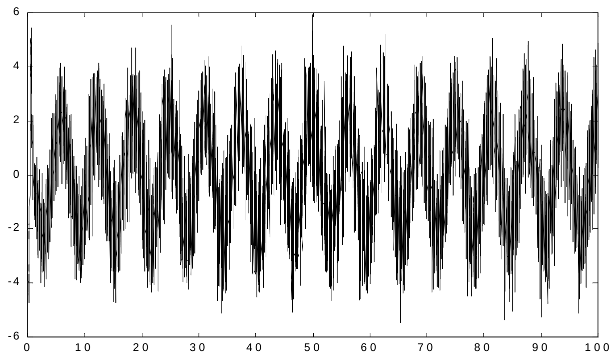

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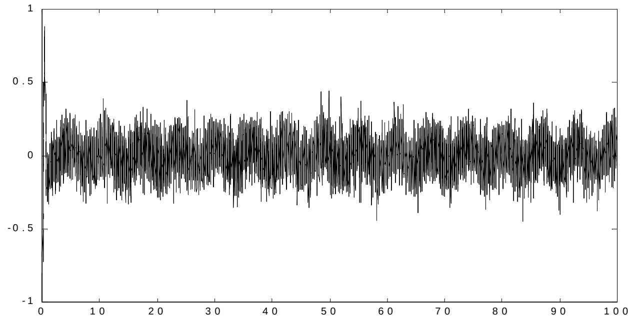

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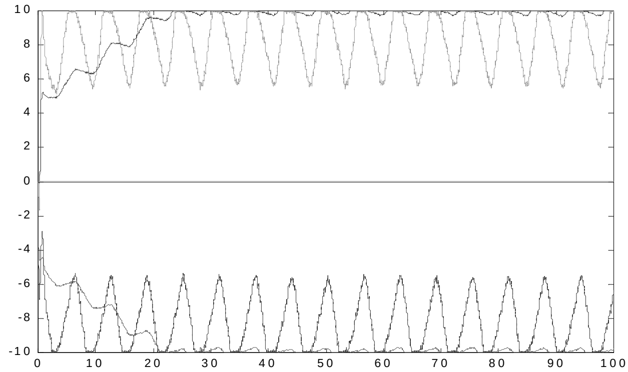
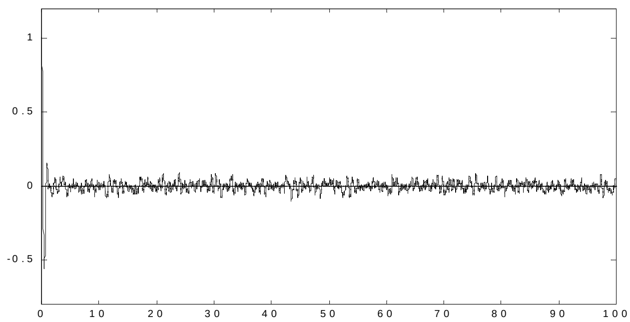


(d)

Figure 8 (a) The actual output *y*, (b) the control *u*, (c) the error *e*, and (d) the six θ of the pure controller with $r_n = 0.1randn(1)$, *P*=100, and $M_\theta = \pm 10$.



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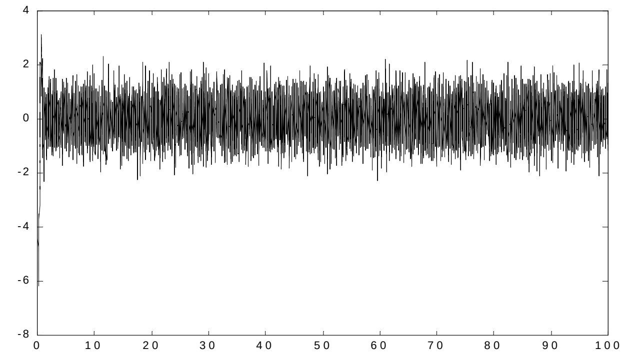

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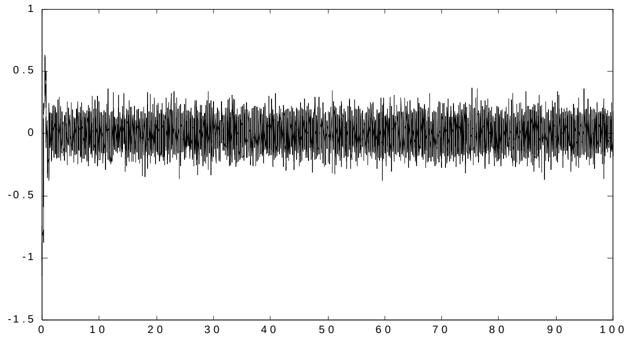

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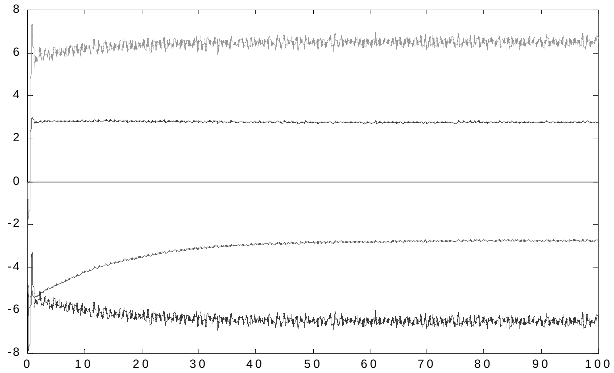
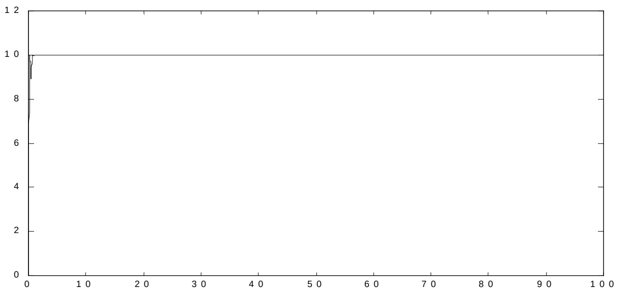
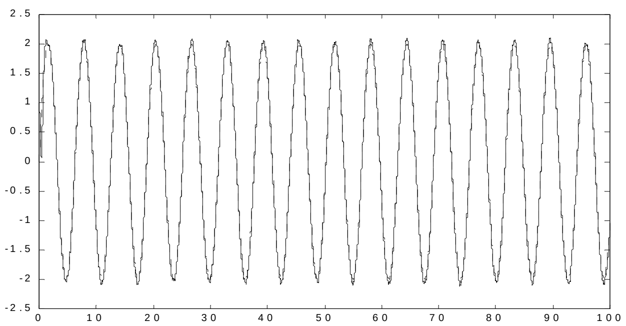


(d)

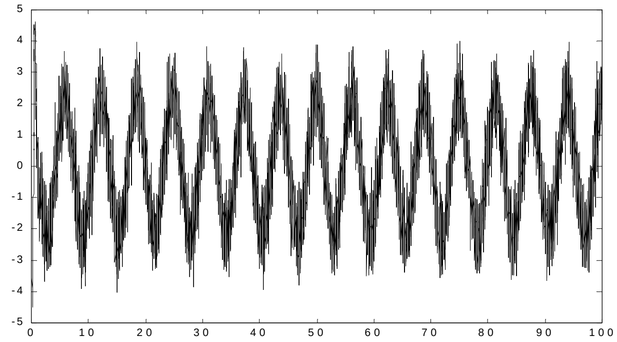


(e)

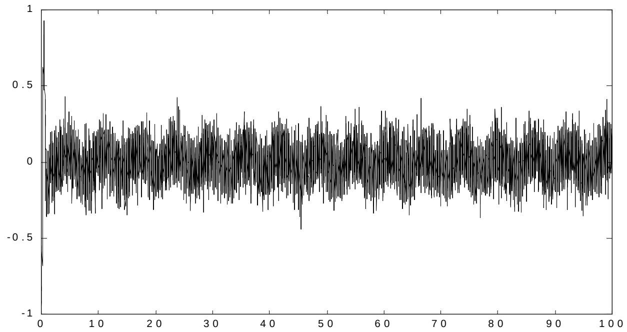
Figure 9 (a) The actual output *y*, (b) the control *u*, (c) the error *e*, (d) the six parameters θ of the pure controller, and (e) the norm of the all parameters θ with $r_n = 0.1randn(1)$, *P*=100, and $M_{n\theta} = \pm 10$.



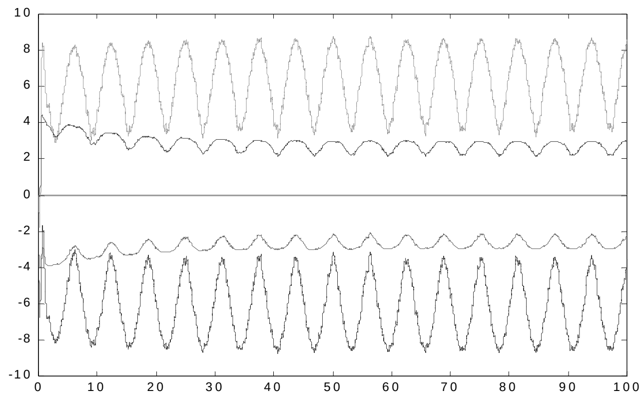
(a)



(b)



(c)



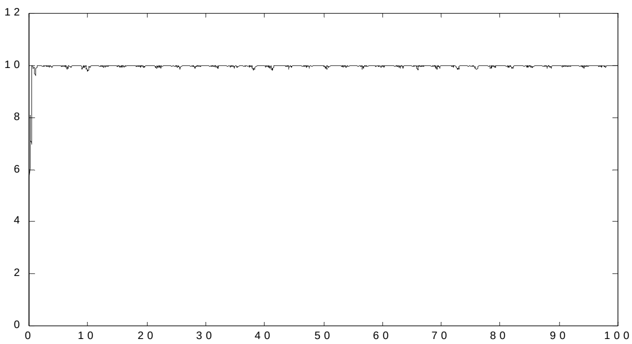


Figure 10 (a) The actual output *y*, (b) the control *u*, (c) the error *e*, (d) the six θ of the pure controller, and 10 (e) The norm of the all parameters θ with $r_n = 0.1randn(1)$, *P*=100, and $M_{n\theta} = \pm 10$.

From the above, it is evident that when the tuned parameters are bounded as shown in Fig. 7 (d) and 8 (d) or in Fig. 9 (d) and 10 (d), the control *u* can be bounded as shown in Fig. 7 (b), 8 (b), 9 (b), and 10 (b). As a result, the actual output *y* of the system converges as shown in Fig. 7 (a), 8 (a), 9 (a) and 10 (a). The sums of the absolute errors of the above regulation cases (as shown in Fig. 5 (c), 7 (c), and 9 (c)) are 910.5869, 864.3808, and 855.1192, respectively. Table 2 lists the comparisons of the sums of the absolute errors of Fig. 6 (c), 8 (c), and 10 (c), by cycles. From those results, it can be concluded that the approach of using bounds for the norms of the tuned parameters can have better performances.

Now with constraints in the tuned parameters, the performances of using different values for *P* are studied. Here, we use the bound for the norm of the tuned parameters. Table 3 shows the results for the regulation case. The smallest error is observed when *P*=100. Table 4 shows the results of the tracking case. The smallest error is observed when *P*=500. From Tables 3 and 4, it can be found that when *P* is too large or too small, the resultant errors both are large. When there is no sensory noise, the larger is *P*, the better is the performance. When there exist noise, a large *P* may result in a large amount of tuning but can have fast convergent speed in the beginning stage. When *P* is small, although there is a small error in the adaptive process, the convergent speed is slow in the beginning stage. Thus, the parameter *P* must be selected appropriately in real applications.

4. Conclusions

In this paper, the direct adaptive fuzzy controller is studied. The adaptive controller is designed based on the Lyapunov synthesis approach. In this paper, we discussed the variation of a parameter required in the Lyapunov method. We found that the parameter matrix *P* must be selected appropriately beside of satisfying the symmetric positive definite property. In our simulations, it can be found that the controller does not converge to a fixed value but varies with trajectories. If we use the original controller for an unstable system with sensory noise, the output error will always drive the control unbounded. In our study, two ways of restraining controllers are employed to resolve this problem. Simulations show that both approaches can resolve the unstable problem. Hopefully, with our discussion and research, some insights for the design of direct adaptive fuzzy controllers can be noticed. We believe that there are some new ideas that can provide hints for developing new approaches for adaptive fuzzy controllers.

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